Axioms of probability :

Imp The probability of an event A denoted by $P(A)$ is chosen as to satisfy Three Axioms.

(i) $P(A) \geq 0$

For an event A in a random experiment the probability may be zero (or) any positive number. It must not be -ve numb.

(ii) $P(S) = 1$

for a sample space 'S' itself and event comprising all possible outcomes the highest possible probability is one.

(iii) $P(A \cap B) = 0$, Then $P(A \cup B) = P(A) + P(B)$

If, A & B are two mutually exclusive events the possible outcome of A & B is zero. That is $P(A \cap B) = 0$.

* Prove that $P(A') = 1 - P(A)$

Imp $P(A') = 1 - P(A)$

$\because$ A and A' are mutually exclusive events.

Then $P(A \cap A') = 0$ [$\because$ By Axiom 3

$P(A \cup A') = P(A) + P(A') - P(A \cap A')$

$= P(A) + P(A') - 0$

$P(A \cup A') = P(A) + P(A')$ [$\because A \cup A' = S$]

$P(S) = P(A) + P(A')$

$P(A) + P(A') = 1$

$P(A') = 1 - P(A)$

Theorem-2

Imp For any two events a & b

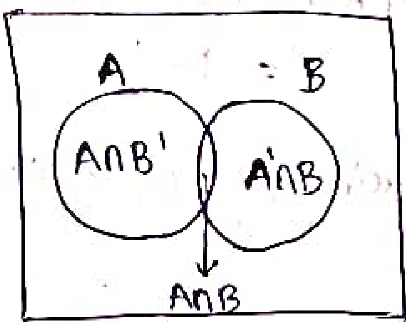
(i) $P(A' \cap B) = P(B) - P(A \cap B)$

(ii) $P(A \cap B') = P(A) - P(A \cap B)$

(i) By using venn diagram

$B = (A \cap B) \cup (A' \cap B)$

$P(B) = P[(A \cap B) \cup (A' \cap B)]$



$$P(B) = P(A \cap B) + P(A' \cap B)$$

[$\because$ $A \cap B$ and $A' \cap B$ are Two mutually exclusive Events

By Axiom 3 : $P[(A \cap B) \cap (A' \cap B)] = 0$

$$P(A' \cap B) = P(B) - P(A \cap B)$$

(ii) By using Venn-Diagram

$$A = (A \cap B) \cup (A \cap B')$$

$$P(A) = P[(A \cap B) \cup (A \cap B')]$$

$$P(A) = P(A \cap B) + P(A \cap B')$$

[$\because$ $A \cap B$ and $A \cap B'$ are Two mutually exclusive Events.]

By Axiom 3 : $P[(A \cap B) \cap (A \cap B')] = 0$

$$P(A \cap B') = P(A) - P(A \cap B)$$

Theorem-3:

Theorem: $P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C) + P(A\cap B\cap C)$

Imp

Proof: Let $B\cup C = D$

$$P(A\cup D) = P(A)+P(D)-P(A\cap D)$$
$$= P(A)+P(B\cup C)-P[A\cap(B\cup C)]$$
$$= P(A)+P(B)+P(C)-P(B\cap C)-P[(A\cap B)\cup(A\cap C)]$$
$$= P(A)+P(B)+P(C)-P(B\cap C)-[P(A\cap B)+P(A\cap C)-P(A\cap B\cap C)]$$
$$= P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$$

Statement: Additional Theorem for n events:

Theorem: For n events $A_1, A_2, A_3 \ldots A_n$

$$\bigcup_{i=1}^{n} P(A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i\le j=1}^{n}\sum P(A_i\cap A_j) + \cdots + (-1)^n \sum_{i=1}^{n-1} P(A_1\cap A_2 \ldots A_i)$$

Imp

$$+ (-1)^{n-1} \bigcap_{i=1}^{n} P(A_i)$$

- $\bigcup_{i=1}^{n} P(A_i) = P(A_1\cup A_2 \ldots A_n)$
- $\sum_{i=1}^{n} P(A_i) = P(A_1)+P(A_2)+\ldots P(A_n)$
- $\sum_{i\le j=1}^{n}\sum P(A_i\cap A_j) = P(A_1\cap A_2)+P(A_2\cap A_3)+\ldots+P(A_{n-1}\cap A_n)$
- $\bigcap_{i=1}^{n} P(A_i) = P(A_1\cap A_2\cap \ldots A_n)$

Proof: Given $A_1, A_2, \ldots A_n$ are 'n' events.

By the method of mathematical Induction

Let us take $n=2$ events i.e., A_1 and A_2

Then

$$\text{LHS} = \bigcup_{i=1}^{2} P(A_i) = P(A_1\cup A_2)$$
$$= P(A_1)+P(A_2)-P(A_1\cap A_2)$$
$$= \sum_{i=1}^{2} P(A_i) - \bigcap_{i=1}^{2} P(A_i)$$
$$= \text{RHS}$$

$\therefore$ It is True for $n=2$.

If $n=3$

$$\bigcup_{i=1}^{3} P(A_i) = P(A_1\cup A_2\cup A_3)$$
$$= P(A_1)+P(A_2)+P(A_3)-P(A_1\cap A_2)-P(A_2\cap A_3)-P(A_1\cap A_3) + P(A_1\cap A_2\cap A_3)$$

$$= \sum_{i=1}^{3} P(A_i) - \sum\sum_{i\le j=1}^{3} P(A_i \cap A_j) + (-1)^{3-1} \bigcap_{i=1}^{3} P(A_i)$$

$\therefore$ It is True for $n=3$

let us suppose that it is true for $n=r$ ie.,

$$\bigcup_{i=1}^{r} P(A_i) = \sum_{i=1}^{r} P(A_i) - \sum\sum_{i\le j=1}^{r} P(A_i \cap A_j) + (-1)^{r-1} \bigcap_{i=1}^{r} P(A_i) \rightarrow ①$$

Now to prove that $n=r+1$ is true.

$$\bigcup_{i=1}^{r+1} P(A_i) = \bigcup_{i=1}^{r} P[A_i \cup A_{r+1}]$$

$$= \left[\sum_{i=1}^{r} P(A_i) - \sum\sum_{i\le j=1}^{r} P(A_i \cap A_j) + \dots + (-1)^{r-1} \bigcap_{i=1}^{r} P(A_i)\right] \cup P(A_{r+1})$$

$$\bigcup_{i=1}^{r+1} P(A_i) = P\left[\bigcup_{i=1}^{r} A_i \cup A_{r+1}\right]$$

$$= P \bigcup_{i=1}^{r} A_i + P(A_{r+1}) - P\left[\bigcup_{i=1}^{r} A_i \cap A_{r+1}\right]$$

$$= \sum_{i=1}^{r} P(A_i) - \sum\sum_{i\le j=1}^{r} P(A_i \cap A_j) + \dots + (-1)^{r-1} \bigcap_{i=1}^{r} P(A_i) + P(A_{r+1})$$

$$- P\left[\bigcup_{i=1}^{r} P(A_i \cap A_{r+1})\right]$$

$$= \sum_{i=1}^{r} P(A_i) + P(A_{r+1}) - \sum\sum_{i\le j=1}^{r} P(A_i \cap A_j) + \dots (-1)^{r-1} \bigcap_{i=1}^{r} P(A_i) -$$

$$\left[\sum\sum_{i\le j=1}^{r} P(A_i \cap A_{r+1}) + \dots (-1)^{r-1+1} \bigcap_{i=1}^{r+1} P(A_i)\right]$$

$\rightarrow$ by eq ②

Conditional probability :

Imp If A and B are two events, the probability of the outcome B given in A is known as conditional probability (or) Transistion probability.

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \text{for } P(A) > 0$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \text{for } P(B) > 0$$

Continuation of above Theorem (previous)

$$= \sum_{i=1}^{r+1} P(A_i) - \sum\sum_{i\le j=1}^{r} P(A_i \cap A_j) - \sum\sum_{i\le j=1}^{r} P(A_i \cap A_{r+i}) + \dots + (-1)^{r-1}$$

$$\bigcap_{i=1}^{r} P(A_i) - (-1)^{r} \bigcap^{r+1} P(A_i)$$

$$= \sum_{i=1}^{r+1} P(A_i) - \sum\sum_{i\le j=1}^{r+1} P(A_i \cap A_j) + \dots (-1)^{r} \bigcap_{i=1}^{r+1} P(A_i)$$

$\therefore$ It is true for $n = r+1$

Then it is true for

$$\bigcup_{i=1}^{n} P(A_i) = \sum_{i=1}^{n} P(A_i) - \sum\sum_{i\le j=1}^{n} P(A_i \cap A_j) + \dots + (-1)^{n} \sum_{i=1}^{n-1} P(A_1 \cap A_2 \cdots A_i)$$

$$+ (-1)^{n-1} \bigcap_{i=1}^{n} P(A_i).$$

$\because \bigcup_{i=1}^{r} A_i \cap A_{r+1}$

$[(A_1 \cup A_2 \cup \dots A_r) \cap (A_{r+1})]$

$(A_1 \cap A_{r+1}) \cup (A_2 \cap A_{r+1}) \dots \cup (A_r \cap A_{r+1})$

$\to \bigcup_{i=1}^{r} P(A_i \cap A_{r+1})$

→ A Bag Contains 2 green and 3 black balls. A sample size of 4 balls are made. What is the probability That the sample is in the order of $B_1 G_2 B_3 G_4$

Probability of Taking one Black Ball first, $P(B_1) = \frac{3}{5}$

probability of Taking Next ball green, $P(G_2) = \frac{2}{4}$

probability of Taking 3rd Ball Black, $P(B_3) = \frac{2}{3}$

Probability of Taking 4th Ball Green, $P(G_4) = \frac{1}{2}$

The probability of getting an order $B_1 G_2 B_3 G_4$ is, $P(B_1 G_2 B_3 G_4) =$

$$P(B_1) \cdot P\left(\frac{G_2}{B_1}\right) P\left(B_3/B_1 \cdot G_2\right) P\left(G_4/B_1 G_2 B_3\right)$$

$$= \frac{\cancel{3}}{5} \cdot \frac{\cancel{2}}{\cancel{4}} \cdot \frac{2}{\cancel{3}} \cdot \frac{1}{2}$$

$$= \frac{1}{10}$$

$\therefore$ The required probability is $\frac{1}{10}$

$$= \sum_{i=1}^{r+1} P(A_i) - \sum\sum_{i\leq j=1}^{r+1} P(A_i \cap A_j) + \cdots (-1)^{r} \bigcap_{i=1}^{r+1} P(A_i)$$ //

$\therefore$ It is true for $n = r+1$

Then it is true for

$$\bigcup_{i=1}^{n} P(A_i) = \sum_{i=1}^{n} P(A_i) - \sum\sum_{i\leq j=1}^{n} P(A_i \cap A_j) + \cdots + (-1)^{n} \sum_{i=1}^{n-1} P(A_1 \cap A_2 \cdots A_i)$$

$$+ (-1)^{n-1} \bigcap_{i=1}^{n} P(A_i).$$

$\because \bigcup_{i=1}^{r} A_i \cap A_{r+1}$

$[(A_1 \cup A_2 \cup \cdots A_r) \cap (A_{r+1})]$

$(A_1 \cap A_{r+1}) \cup (A_2 \cap A_{r+1}) \cdots \cup (A_r \cap A_{r+1})$

$\rightarrow \bigcup_{i=1}^{r} P(A_i \cap A_{r+1})$

Theorem: For any Three events A, B and C prove that

Imp $P(A\cup B/C) = P(A/C) + P(B/C) - P(A\cap B/C)$.

Proof: We know That

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

Intersect 'C' on Both sides:

$$P((A\cup B)\cap C) = P(A\cap C) + P(B\cap C) - P(A\cap B\cap C)$$

Divide with $P(C)$ on Both sides.

$$\frac{P[(A\cup B)\cap C]}{P(C)} = \frac{P(A\cap C)}{P(C)} + \frac{P(B\cap C)}{P(C)} - \frac{P(A\cap B\cap C)}{P(C)}.$$

[∵ By conditional probability, $P(A/B) = \frac{P(A\cap B)}{P(B)}$]

$$P(A\cup B/C) = P(A/C) + P(B/C) - P(A\cap B/C)$$

Formula: Total probability.

Given That 'n' mutually exclusive events 'B_n'; $n = 1, 2, \ldots n$ whose union's equal to sample space. on the same sample space The probability of any event 'A', $P(A)$ can be written in Terms of conditional probabilities.

ie: $P(A) = \sum_{i=1}^{N} P(A/B_n) \cdot P(B_n)$

Baye's Theorem:

Imp Let $E_1, E_2, \ldots E_n$ be 'n' mutually exclusive disjoint events with $P(E_i) \neq 0$, and $S = \bigcup_{i=1}^{n} E_i$; Then for ~~every~~ any arbitary event A such That $A \subset \bigcup_{i=1}^{n} E_i$ and $P(A) > 0$ Then $P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum P(E_i)\, P(A/E_i)}$

Proof: Given $A \subset \bigcup_{i=1}^{n} E_i$

$$A = A \cap \bigcup_{i=1}^{n} E_i$$

$$A = A \cap [E_1 \cup E_2 \cup E_3 \cup \ldots E_n]$$

$$A = (A\cap E_1) \cup (A\cap E_2) \cup \ldots (A\cap E_n)$$

$$A = \bigcup_{i=1}^{n} (A\cap E_i)$$

$$P(A) = \bigcup_{i=1}^{n} P(A \cap E_i) \rightarrow ①$$

Given $E_1, E_2, \dots E_n$ are 'n' mutually exclusive disjoint events

i.e., $[P(A \cup B) = P(A) + P(B)]$

$\therefore$ from ①, $P(A) = \sum_{i=1}^{n} P(A \cap E_i) \rightarrow ②$.

We know that Conditional probability is

$$P(A/E_i) = \frac{P(A \cap E_i)}{P(E_i)}$$

$$\Rightarrow P(A \cap E_i) = P(A/E_i) \cdot P(E_i) \rightarrow ③$$

From ② & ③, we get

$$\therefore P(A) = \sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)$$

Now LHS $= P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$ (By Conditional probability)

$\therefore$ From ② and ③

$$P(E_i/A) = \frac{P(A/E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(A/E_i) \cdot P(E_i)}$$

$= RHS$

→ State and prove Boole's Inequality?

Imp

<u>Boole's Inequality</u> : For n events $A_1, A_2, \ldots A_n$

(a) $P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$

(b) $P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$

<u>Proof</u> : Given for n events $A_1, A_2, \ldots A_n$

(a) Let us suppose $n=2$ i.e., A_1, A_2

We know That $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1$

$\Rightarrow P(A_1) + P(A_2) - 1 \leq P(A_1 \cap A_2)$

$\Rightarrow P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1 \longrightarrow (1)$

$$\Rightarrow \sum_{i=1}^{2} P(A_i) - 1$$

$\therefore$ It is true for $n=2$

Let us suppose It is true for $n=r$

$$P\left(\bigcap_{i=1}^{r} A_i\right) \geq \sum_{i=1}^{r} P(A_i) - (r-1) \rightarrow ②$$

Let us prove it is true for $n=r+1$

$$P\left(\bigcap_{i=1}^{r+1} A_i\right) = P\left(\bigcap_{i=1}^{r} A_i \cap A_{r+1}\right)$$

From ①

$$P\left(\bigcap_{i=1}^{r+1} A_i\right) \geq P\left(\bigcap_{i=1}^{r} A_i\right) + P(A_{r+1}) - 1$$

from ②

$$P\left(\bigcap_{i=1}^{r+1} A_i\right) \geq \sum_{i=1}^{r} P(A_i) - (r-1) + P(A_{r+1}) - 1$$

$$P\left(\bigcap_{i=1}^{r+1} A_i\right) \geq \sum_{i=1}^{r+1} P(A_i) - r + \cancel{1} - \cancel{1}$$

$$P\left(\bigcap_{i=1}^{r+1} A_i\right) \geq \sum_{i=1}^{r+1} P(A_i) - r$$

$\therefore$ It is true for $n=r+1$.

Hence it is true for 'n' events.

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$

(b) Since From statement ①

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$

$[\because P(A') = 1 - P(A)]$

$[P(A' \cap B') = P(A \cup B)']$

$\therefore$ There are n events, Apply 'n' complements on Both sides

$$P(A_1' \cap A_2' \cap \cdots\cdots A_n') \geq [P(A_1') + P(A_2') + \cdots P(A_n')] - (n-1)$$

$$P(A_1' \cap A_2' \cap \cdots\cdots A_n') \geq [(1-P(A_1)) + (1-P(A_2)) + \cdots (1-P(A_n))] - n + 1$$

$$P(A_1' \cap A_2' \cap \cdots\cdots A_n') \geq \cancel{n} - P(A_1) - P(A_2) \cdots\cdots - P(A_n) - \cancel{n} + 1$$

$$P(A_1 \cup A_2 \cup \cdots A_n)' \geqslant 1-[P(A_1)+P(A_2)+\cdots+P(A_n)]$$

$$P(A_1 \cup A_2 \cup \cdots A_n)'-1 \geqslant -[P(A_1)+P(A_2)+\cdots+P(A_n)]$$

$$1-P(A_1 \cup A_2 \cup \cdots A_n)' \leq P(A_1)+P(A_2)+\cdots+P(A_n)$$

$$P(A_1 \cup A_2 \cup \cdots A_n) \leq P(A_1)+P(A_2)+\cdots+P(A_n)$$

$$\bigcup_{i=1}^{n}(A_i) \leq \sum_{i=1}^{n} P(A_i)$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

Ex: A random variable 'x' has the following functional probability

Imp

values of X, x :	0	1	2	3	4	5	6
$P(x)$:	K	3K	5K	7K	9K	11K	13K

Find (i) K (ii) Evaluate $P(X < 4)$, $P(X \geq 5)$

and $P(3 < X \le 6)$

(iii) What is the smallest value of 'x' for which $P(X \le x) > 1/2$

Sol: (i) From the Axiom (2), $P(S) = 1$

$\Rightarrow \sum_{i=0}^{6} P(x_i) = 1$

$\Rightarrow K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$\Rightarrow 49K = 1$

$\Rightarrow K = \frac{1}{49}$

(ii) $P(X < 4) = P(0) + P(1) + P(2) + P(3)$

$= K + 3K + 5K + 7K$

$= 16K$

$= 16\left(\frac{1}{49}\right) = \frac{16}{49}.$

$P(X \ge 5) = \cancel{P(0) + P(1) + P(2) + P(3) + P(4)} + P(5) + P(6)$

$= \cancel{K + 3K + 5K + 7K + 9K} + 11K + 13K$

$= \cancel{36K}$ $= 24K.$

$= \cancel{36\left(\frac{1}{49}\right)} = \cancel{\frac{36}{49}}.$ $= 24\left(\frac{1}{49}\right) = \frac{24}{49}$

$P(3 < X \le 6) = P(4) + P(5) + P(6)$

$= 9K + 11K + 13K = 33K$

$= 33\left(\frac{1}{49}\right) = \frac{33}{49}$

(iii) The minimum value of 'K' by substituting the values of 'x'

$P(X \le x)$

$P(X \le 0) = \frac{1}{49}$

$P(X \le 1) = \frac{1}{49} + \frac{3}{49} = \frac{4}{49}$

$P(X \le 2) = \frac{1}{49} + \frac{3}{49} + \frac{5}{49} = \frac{9}{49}$

$P(X \le 3) = \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49} = \frac{16}{49}$

$P(X \le 4) = \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49} + \frac{9}{49} = \frac{25}{49}.$

$P(X \le 5) = \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49} + \frac{9}{49} + \frac{11}{49} = \frac{36}{49}$

$P(X \le 6) = \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49} + \frac{9}{49} + \frac{11}{49} + \frac{13}{49} = \frac{49}{49} = 1.$

The smallest value of 'x' for which $P(X \le x) > \frac{1}{2}$ is $x = 4$.

* Verify The following is a distribution function:

Imp

$$F(x) = \begin{cases} 0 & , x < -a \\ \frac{1}{2}\left(\frac{x}{a}+1\right) & , -a \le x \le a \\ 1 & , x > a \end{cases}$$

$F(x)$ is a distribution function only if $f(x)$ is a density function. ie, $f(x) = \frac{d}{dx} F(x)$

$$= \frac{d}{dx}\left[\frac{1}{2}\left(\frac{x}{a}+1\right)\right]$$

$$= \frac{1}{2} \cdot \frac{1}{a} = \frac{1}{2a}$$

$$\therefore \int_{-\infty}^{+\infty} f(x)\, dx = 1$$

$$\Rightarrow \int_{-a}^{+a} \frac{1}{2a}\, dx = \frac{1}{2a} \int_{-a}^{a} dx$$

$$= \frac{1}{2a}\left[x\right]_{-a}^{+a}$$

$$= \frac{1}{2a}\left[a-(-a)\right]$$

$$= \frac{1}{2a}(2a)$$

$$= 1$$

$\therefore$ It is a distribution Function.

UNIT-I

(1) The length of time (in minutes) that a certain lady speaks on the telephone is found to be random phenomenon, with a probability function specified by the function

$$f(x) = \begin{cases} A e^{-x/5} & , \text{for } x \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

(a) Find the value of A that makes $f(x)$ a p.d.f.

(b) What is the probability that the number of minutes that she will talk over the phone is

(i) More than 10 minutes

(ii) less than 5 minutes and

(iii) between 5 and 10 minutes.

(a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ for a valid p.d.f (probability distribution function)

$\Rightarrow \int_{0}^{\infty} A e^{-x/5}\,dx = 1$

$\Rightarrow A\left[\frac{e^{-x/5}}{-1/5}\right]_0^{\infty} = 1$

$\Rightarrow -5A\left[e^{-x/5}\right]_0^{\infty} = 1$

$\Rightarrow -5A\left[e^{-\infty} - e^{0}\right] = 1$

$\Rightarrow -5A[-1] = 1$

$\therefore A = 1/5$.

(b) (i) we have $f(x) = \begin{cases} \frac{1}{5} e^{-x/5} & , x \geq 0 \\ 0 & , \text{otherwise} \end{cases}$

The probability that the lady talks more than 10 minutes over the phone is given by

$$P[x > 10] = \int_{10}^{\infty} \frac{1}{5} e^{-x/5}\,dx$$

$$= \frac{1}{5}\left[\frac{e^{-x/5}}{-1/5}\right]_{10}^{\infty} = -\left[e^{-\infty} - e^{-2}\right]$$

$= e^{-2}$

$= \frac{1}{e^2}$

(ii) The probability that the lady talks less than 5 minutes over the phone is given by

$$P(x<5) = 1-P(X\geq 5)$$

$$= 1-\int_5^\infty \frac{1}{5} e^{-x/5} dx$$

$$= 1-\left[\frac{1}{5}\left(\frac{e^{-x/5}}{-1/5}\right)_5^\infty\right]$$

$$= 1+(e^{\infty}-e^{-1})$$

$$= 1-e^{-1}$$

(iii) The probability that the lady talks between 5 and 10 minutes is given by

$$P(5<X<10) = \frac{1}{5}\int_5^{10} e^{-x/5} dx$$

$$= \frac{1}{5}\left[\frac{e^{-x/5}}{-1/5}\right]_5^{10}$$

$$= -[e^{-2}-e^{-1}]$$

$$= e^{-1}-e^{-2}$$

$$= \frac{1}{e}-\frac{1}{e^2} = \frac{e-1}{e^2}$$

(2) An urn A contains 2 white and 4 Black Balls. Another urn B contains 5 white and 7 Black Balls. A Ball is Transferred from urn A to urn B. Then a ball is drawn from urn B. Find the probability that it will be white.

The Ball Transferred can either be Black or white.

(i) <u>When white Ball is Transferred.</u>

probability of drawing white Ball from urn A = $\frac{2}{6}$

Now urn B has 6 white and 7 Black balls. So probability of drawing a white Ball from urn B is $\frac{6}{13}$.

$\therefore$ probability of the compound Event i.e., trans-ferring a white ball and Then drawing a white Ball $= \frac{2}{6} \times \frac{6}{13}$

$= \frac{2}{13}$.

(ii) When Black ball is Transferred:

probability of drawing black Ball from urn A $= \frac{4}{6}$

Now, urn B has 5 white and 8 Black Balls, so the probability of drawing a white Ball from urn B is $= \frac{5}{13}$.

$\therefore$ Probability of the compound Event i.e., Transferring a Black Ball and Then drawing a white Ball $= \frac{4}{6} \times \frac{5}{13} = \frac{10}{39}$.

The Two Events are mutually exclusive Hence, the probability of Trans-ferring a ball from urn A to B and Then drawing a white ball from B $= \frac{2}{13} + \frac{10}{39} = \frac{16}{39}$.

(3) prove That if $B \subset A$, Then $P(A \cap B') = P(A) - P(B)$.

When $B \subset A$, The Events B and $A \cap B'$ are disjoint (mutually Exclusive)

we have $B \cup (A \cap B') = A$

$$P(B) + P(A \cap B') = P(A)$$

Therefore, $P(A \cap B') = P(A) - P(B)$

(1) A card is drawn from a deck of 52 cards.

(a) What is the probability that a 2 is drawn.

(b) What is the probability that a 2 of clubs is drawn.

(c) What is the probability that a spade is drawn.

(2) Four persons write their names on individual slips of paper and deposit the slips in a common BOX. Each of the four draws at random a slip from the BOX. Determine the probability of each person drawing his own name slip.

(3) From a deck of 52 cards, four cards are drawn. If A_1, A_2, A_3 and A_4 are Events of drawing a king on the First, Second, Third and Fouth cards respectively. Find the Total probability if (a) Each card is replaced after the draw (b) not replaced.

15. For any two events A and B, if $P(A) = P_1$, $P(B) = P_2$ and $P(A \cap B) = P_3$, prove the following:

(i) $P(A' \cup B') = 1 - P_3$

(ii) $P(A' \cap B') = 1 - P_1 - P_2 + P_3$

(iii) $P(A \cap B') = P_1 - P_3$

(iv) $P(A' \cap B) = P_2 - P_3$

(v) $P[(A \cap B)'] = 1 - P_3$

(vi) $P(A \cup B') = 1 - P_1 + P_3$

(vii) $P[A' \cap (A \cup B)] = P_2 - P_3$

(viii) $P[A \cup (A' \cap B)] = P_1 + P_2 - P_3$

(ix) $P(A/B) = \dfrac{P_3}{P_2}$

(x) $P(B/A) = \dfrac{P_3}{P_1}$

(xi) $P(A'/B') = \dfrac{1 - P_1 - P_2 + P_3}{1 - P_2}$

(xii) $P(B'/A') = \dfrac{1 - P_1 - P_2 + P_3}{1 - P_1}$